Cosmological perturbations during the Bose-Einstein condensation of dark matter

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Abstract

In the present work, we analyze the evolution of the scalar and tensorial perturbations and the quantities relevant for the physical description of the Universe, as the density contrast of the scalar perturbations and the gravitational waves energy density during the Bose-Einstein condensation of dark matter. The behavior of these parameters during the Bose-Einstein phase transition of dark matter is analyzed in details. To study the cosmological dynamics and evolution of scalar and tensorial perturbations in a Universe with and without cosmological constant we use both analytical and numerical methods. The Bose-Einstein phase transition modifies the evolution of gravitational waves of cosmological origin, as well as the process of large-scale structure formation.

1 Introduction

In what concerns the standard cosmological cenarium (ΛCDM model) the observational data [1, 2] show that the ordinary baryonic matter is responsible for approximately only 5 % (neglecting radiation) of the total energy density of the Universe. Nearly 70% of the total density is the contribution of the so called dark energy (DE), responsible for the observed acceleration of the cosmic expansion [3, 4]. The other 25% is in the form of a weakly interacting matter, called cold dark matter (CDM) or simply dark matter (DM), whose properties are inferred from its gravitational interaction with visible matter, radiation and its effect on the large scale structure of the Universe.

The existence of DM was initially observed due to discrepancies between the calculated mass of the observed matter and the mass determined from the gravitational effects. Subsequently the observation of the rotational curves of galaxies, the velocity dispersions of galaxies, the gravitational lenses and the large scale structures [5, 6] suggested the existence of the DM. There are several hypothetical candidates for DM particles *e.g.* weakly interacting massive particles (WIMPs), axions, hidden dark matter, sterile neutrinos and light gravitinos [7] and although the nature of DM is still unkown a great effort has been made in an attempt to detect the DM particles [5, 6, 7].

The BEC process, that is a very well observed phenomenon in terrestrial experiments, occurs when a gas of bosons is cooled at very low temperatures, near absolute zero, what makes a large fraction of the particles occupy the same ground state. This kind of condensation could have occured at

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1

some moment during the cosmic history of the Universe. The cosmic BEC mechanism was broadly discussed in [8, 9]. In general the BEC takes place when the gas temperature is below the critical temperature $T_{\text{crt}} < 2\pi\hbar^2 n^{2/3}/mk_B$, where n is the particles density, m is the particle mass and k_B is the Boltzmann's contant. Since in an adiabatic process a matter dominated Universe behaves as $\rho \propto T^{3/2}$ the cosmic dynamics has the same temperature dependence. Hence we will have the critical temperature at present $T_{\text{crt}} = 0.0027$ K if the boson temperature was equal to the radiation temperature at the red-shift $z = 1000$ [9]. During the cosmic adiabatic evolution the ratio of the photon temperature and the matter temperature evolves as $T_{\text{r}}/T_{\text{m}} \propto a$, where a is the scale factor of the Universe. Using as value for the present energy density of the Universe $\rho = 9.44 \times 10^{-30}\text{g/cm}^3$ BEC will happens if the boson mass satisfies $m < 1.87\text{eV}$.

Despite the great sucess and simplicity of the standard ΛCDM cosmological model, it still remains as a phenomenological model and we can not explain why the cosmological constant density has a value close to the critical density today and why the measured cosmological constant is smaller than the quantum vacuum energy density by a factor of 10^{-120}. At galactic scales, this model has difficulties in explaining the observed distribution of DM around the luminous one. The simulations in the ΛCDM model predict that bound halos arround galaxies must have very characteristic density profiles with a well pronounced central cusp [10], but the observations of rotation curves show that the distribution of DM is shallower than the predicted and it has a approximately constant density core [11]. The so called core-cusp problem can be solved by presuming that DM particles are ultralight scalar particles with masses of the order of 10^{-22} eV initially in a cold BEC [12].

Assuming the hypothesis that the CDM in a galaxy is in form of BEC the density distribution of the static gravitationally bounded single component BEC DM is given by $\rho(r) = \rho_* \sin kr/kr$, where $\rho_* = \rho(0)$ is the density in the center of the condensate and k is a constant. Giving the conditions $\rho(R) = 0$ and $kR = \pi$, where R is the condensate radius, the condensate DM halo radius can be fixed as $R = \pi(\hbar^2 l_s/Gm^3)^{1/2}$ [13] where l_s is the particle scattering length and m is the the particle mass. The calculated total mass of the condensate DM halo is $M = 4\pi^2(\hbar^2 l_s/Gm^3)^{3/2}\rho_* = 4R^3\rho_*/\pi$ [13]. So the mass of the particles of the condensate are [13]

$$m = \left(\frac{\pi^2\hbar^2 l_s}{GR^2}\right)^{1/3} \approx 6.73 \times 10^{-2}\left(\frac{l_s}{1\text{ fm}}\right)^{1/2}\left(\frac{R}{1\text{ kpc}}\right)^{-2/3}\text{ eV} \quad . \tag{1}$$

Recently the cosmological BEC process of DM was investigated and in this model [14] it is assumed that the condensation process is a phase transition that occurs at some time during the history of the Universe. In this model the normal bosonic DM cools below the critical condensation temperature, that turns to be favorable to form the condensate in which all particles occupy the same ground state. In this new period the two phases coexist for some time until all ordinary DM is converted into the condensated form, when the transition ends. The time evolution of cosmological parameters as the energy density, temperature and scale factor is changed during the phase transition process [14].

In this letter we aim to investigate how the generation of cosmological perturbations produced by the space-time expansion, and the parameter associated with them, would be changed during this Bose-Einstein phase transition of dark matter. The scalar perturbations are related to the formation of structures like galaxy clusters and galaxy superclusters in the Universe. The generally accepted picture of the structure formation in cosmology is the one where the Universe started off in an extremely homogeneous and isotropic state, with initial conditions provided by an era of accelerated expansion called inflation. The tiny primordial density fluctuations, generated during inflation from quantum fluctuations of the vacuum, would later grow under the influence of gravity and eventually collapse to form the structures that we observe today. In general, the evolution of the density of the different field components of the matter are calculated analytically or numerically depending on the complexity of

the studied model. These calculations are made in such a way that they may be compared with the observations of large-scale structure of the Universe like, *e.g.*, Cosmic Microwave Background (CMB) anisotropy maps derived from WMAP data [1, 2], Type Ia supernova (SNIa) surveys [3, 4] and the baryon acoustic oscillations (BAO) in the Sloan Digital Sky Survey (SDSS) luminous galaxy sample [15].

The gravitational waves are tensorial fluctuations in the metric of space-time. This particular perturbation is not explicitly coupled with the energy density and pressure of the matter of the Universe and does not contribute to the gravitational instability that generates the cosmological structures we see today. On the other hand this study is of great interest because it supplies the specific signature of the metric theory of gravity. These waves are predicted by Einstein's theory of General Relativity (GR) but they still have to be directly detected. Great efforts have been done in this sense and there is hope that a new generation of experiments in space may allow this detection [16]. The gravitational waves spectrum frequency extends over a wide range of interest, from $10^{-18} Hz$ to $10^8 Hz$, depending on the sources that generate those waves.

The present letter is organized as follows. The basic properties of the Bose-Einstein condensate dark matter (BEC DM) and the ordinary dark matter are reviewed in Section 2. The cosmological dynamics of the density perturbations is considered in Section 3. The evolution of the gravitational waves is derived in Section 4. We discuss and conclude our results in Section 5.

2 Ordinary Dark Matter and Bose-Einstein condensated Dark Matter

Following the present data [1, 2] we assume a flat homogeneous and isotropic Universe, whose geometry is described by the Friedmann-Robertson-Walker metric, given by

$$ds^2 = c^2 dt^2 - a(t) d\vec{x}^2 \quad , \tag{2}$$

where $a(t)$ is the scale factor of the Universe that describes the cosmic evolution, t is the cosmic time and c is the speed of light. The gravitational dynamics is given by the Einstein field equations

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4} T_{\mu\nu} \quad . \tag{3}$$

We also consider the Universe filled by a perfect fluid, described by the energy-momentum tensor

$$T^{\mu\nu} = (\rho c^2 + p) u^\mu u^\nu - p g^{\mu\nu} \quad , \tag{4}$$

where ρ is the density of the fluid, p is the pressure and $g^{\mu\nu}$ is the metric tensor.

Imagining the Universe filled by baryonic matter, radiation, dark matter and cosmological constant and neglecting any possible interaction between those components by assuming that the energy of each component is individually conserved, the equations of motion are

$$\frac{\dot{a}^2}{a^2} = \frac{8\pi G}{3}(\rho_b + \rho_r + \rho_\chi + \rho_\Lambda) \quad , \tag{5}$$

$$2\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} = -\frac{8\pi G}{c^2}(p_r + p_\chi + p_\Lambda) \quad , \tag{6}$$

$$\dot{\rho}_i + 3\left(\rho_i + \frac{p_i}{c^2}\right)\frac{\dot{a}}{a} = 0 \quad . \tag{7}$$

where $H = \dot{a}/a$ is the Hubble parameter and $i = b,\ r,\ \chi,\ \Lambda$ means baryonic matter, radiation, DM and comological constant, respectively.

3

2.1 Non-condensated Dark Matter

We will presume that in the early phases of the Universe the DM was made of bosons particles with mass m_χ and temperature T, originated in equilibrium and decoupled at a temperature T_D. Particles that decouple from the plasma in the primordial Universe at any temperature T_D has its momenta redshifted such that the distribution function at any time after de decoupling is related to the value of the distribution function at the moment of the decoupling and it keeps an equilibrium shape in both non-relativistic and extreme-relativistic regimes [14]. In the non-relativistic regime the non-condensated DM pressure is [14]

$$p_\chi = c^2 \sigma^2 \rho_\chi \quad , \tag{8}$$

where $\sigma^2 = \langle \vec{v}^{\,2} \rangle / 3c^2$ is the velocity dispersion and $\langle \vec{v}^{\,2} \rangle$ is the mean square velocity of the particle. Appling the equation of state (8) into the conservation equation (7) leads to

$$\rho_\chi = \frac{\rho_\chi^{(0)}}{(a/a_0)^{3(1+\sigma^2)}} \quad , \tag{9}$$

where $\rho = \rho_\chi^{(0)}$ when $a = a_0$. Replacing the evolution equation (9) for the non-condensated DM into the Friedmann's equation (5) we find that the evolution of a Universe filled by baryonic matter, radiation, non-condensated DM and cosmological constant is given by

$$\frac{\dot{a}^2}{a^2} = H_0^2 \left(\frac{\Omega_b^{(0)}}{(a/a_0)^3} + \frac{\Omega_r^{(0)}}{(a/a_0)^4} + \frac{\Omega_\chi^{(0)}}{(a/a_0)^{3(1+\sigma^2)}} + \Omega_\Lambda \right) \quad , \tag{10}$$

where H_0 is the Hubble parameter and $\Omega_i^0 = 8\pi G \rho_i^{(0)} / 3H_0^2$ is the density parameter, both at $a = a_0$. The density parameters satisfy the relation $\Omega_b^{(0)} + \Omega_r^{(0)} + \Omega_\chi^{(0)} + \Omega_\Lambda = 1$ and a_0 can be considered the present day scale factor. Since the DM is non-relativistic the cosmic evolution is little dominated by the term σ^2. In the standard model $\sigma^2 = 0$.

2.2 Bose-Einstein condensated Dark Matter

The BEC is formed when all particles in a Bose gas occupy the same quantum ground state. This happens at a very low critical temperature $T_{\text{crt}} \approx 2\pi\hbar^2 \rho^{2/3}/m^{5/3}k_B$ [8, 9, 14], where m is the mass of the particles, ρ is the density and k_B is the Boltzmann's constant. The coherent state is reached when the temperature is low enough or the density is sufficiently high [14]. In this model we will assume that almost all the DM particles are in the condensate form and that the DM halos are made of BEC DM at absolute zero temperature.

Only collisions between two particles at low energy are relevants in a diluate and cold gas. These collisions are characterized by the scattering length l_s independently of the two-body potential [14], hence the interaction potencial can be replaced by an effective interaction $V_I(\vec{r}\,' - \vec{r}) = \lambda\delta(\vec{r}\,' - \vec{r})$ [17, 14] where $\lambda = 4\pi\hbar^2 l_s/m$. The ground state features of BEC DM halos is described by the generalized Gross-Pitaevskii (GP) equation [17, 14]

$$i\hbar\frac{\partial\phi(t,\vec{r})}{\partial t} - \frac{\hbar^2}{2m}\nabla^2\phi(t,\vec{r}) + mV(\vec{r})\phi(t,\vec{r}) + U_0|\phi(t,\vec{r})|^2\phi(t,\vec{r}) \quad , \tag{11}$$

where $\phi(t,\vec{r})$ is the wave function of the condensate, m is the particles mass, V is tha gravitational potential that satisfies the Poisson's equation $\nabla^2 V(\vec{r}) = 4\pi G\rho$ and $U_0 = 4\pi\hbar^2 l_s/m$. We can use the Madelung representation of the wave function [8, 9, 17], which is

$$\phi(t,\vec{r}) = \sqrt{\rho(t,\vec{r})} \times e^{iS(t,\vec{r})/\hbar} \quad , \tag{12}$$

where $\rho(t, \vec{r}) = |\phi(t, \vec{r})|^2$ is the density of the condensate and the function $S(t, \vec{r})$ has the dimension of an action. This transformation will make the generalized GP equation (11) breaks into two equations

$$\frac{\partial \rho_\chi}{\partial t} + \nabla \cdot (\rho_\chi \vec{v}) = 0 \quad , \tag{13}$$

$$\rho_\chi \left(\frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \nabla) \vec{v} \right) = -\nabla p_\chi \left(\frac{\rho_\chi}{m} \right) - \rho_\chi \nabla \left(\frac{V}{m} \right) - \nabla V_Q \quad , \tag{14}$$

where $V_Q = -(\hbar^2/2m)\nabla^2 \sqrt{\rho_\chi}/\rho_\chi$ is a quantum potential and $\vec{v} = \nabla S/m$ is the velocity of the quantum fluid. The effective pressure of the condensate [8, 9, 14] is given by

$$p_\chi \left(\frac{\rho_\chi}{m} \right) = \frac{2\pi\hbar^2 l_s}{m^3} \rho_\chi^2 \quad , \tag{15}$$

and the dynamics of the fluid is determined by the scatering lenth l_s and the mass m of the DM particles. Using the conservation equation (7) and the equation of state (15) we find

$$\dot{\rho}_\chi + 3\rho_\chi (1 + \omega_\chi \rho_\chi) \frac{\dot{a}}{a} = 0 \quad , \tag{16}$$

with $\omega_\chi = 2\pi\hbar^2 l_s/m^3 c^2$. The evolution of the BEC DM density is given by

$$\rho_\chi = \frac{C}{(a/a_0)^3 - \omega_\chi C} \quad , \tag{17}$$

where C is an integration constant. Ensuring that $\rho_\chi = \rho_\chi^{(0)}$ at $a = a_0$ we will obtain

$$\rho_\chi = \frac{1}{\omega_\chi} \frac{\rho_{0\chi}}{(a/a_0)^3 - \rho_{0\chi}} \quad , \tag{18}$$

with

$$\rho_{0\chi} = \frac{\omega_\chi \rho_\chi^{(0)}}{1 + \omega_\chi \rho_\chi^{(0)}} = \frac{\omega_\chi \rho_{crt}^{(0)} \Omega_\chi^{(0)}}{1 + \omega_\chi \rho_{crt}^{(0)} \Omega_\chi^{(0)}} \quad , \tag{19}$$

where the critical density of the Universe today is $\rho_{crt}^{(0)} = 9.24 \times 10^{-27}$ Kg/m^3. And the Friedmann's equation (5) for the BEC DM is

$$\frac{\dot{a}^2}{a^2} = H_0^2 \left(\frac{\Omega_b^{(0)}}{(a/a_0)^3} + \frac{\Omega_r^{(0)}}{(a/a_0)^4} + \frac{1}{\omega_\chi} \frac{\Omega_{0\chi}}{(a/a_0)^3 - \rho_{0\chi}} + \Omega_\Lambda \right) \quad , \tag{20}$$

where $\Omega_{0\chi} = \rho_{0\chi}/\rho_{crt}^{(0)}$.

2.3 Cosmological dynamics during the condensation phase

The best description of the BEC dynamics seems to be a first order phase transition [14], since it was demonstrated, by analyzing the temperature and density dependence of the chemical potential in several theoretical models that describe the thermodynamical transition from the normal phase to the BEC phase, that none of them predicts a second-order phase transition [14, 18, 19]. On the other hand a first order phase transition characterize a genuine mathematical singularity, but for the case of a ideal Bose gas confined in a cubic box it was shown that a system with a finite number of particles can

5

present a discontinous phase transition that characterize a genuine mathematical singularity, provided that the pressure is kept constant [19].

During the BEC process the first thermodynamic condition that has to be satisfied is the continuity of the pressure at the transition point. So the equations of state (8) and (15) give the critical transition density [14]

$$\rho_\chi^{crt} = \frac{c^2 \sigma^2 m^3}{2\pi \hbar^2 l_s} = \frac{\sigma^2}{\omega_\chi} \quad , \tag{21}$$

the critical temperature [14, 17] is

$$T_{crt} = \frac{2\pi \hbar^2}{\zeta(3/2)^{2/3} m^{5/3} k_B} \left(\rho_\chi^{crt}\right)^{2/3} \quad , \tag{22}$$

where $\zeta(3/2)$ is the Riemann zeta function. The scale factor of the Universe at the moment of the condensation is [14]

$$\frac{a_{crt}}{a_0} = \left(\frac{2\pi \hbar^2 l_s}{c^2 \sigma^2 m^3} \rho_{crt}^{(0)} \Omega_\chi^{(0)} \right)^{\frac{1}{3(1+\sigma^2)}} \quad , \tag{23}$$

and the critical redshift is

$$1 + z_{crt} = \left(\frac{2\pi \hbar^2 l_s}{c^2 \sigma^2 m^3} \rho_{crt}^{(0)} \Omega_\chi^{(0)} \right)^{\frac{-1}{3(1+\sigma^2)}} \quad . \tag{24}$$

During the transition phase the pressure $P = P_{crt}$ and the temperature $T = T_{crt}$ are constant and after the beginning of the phase transition the DM density decreases from ρ_χ^{norm}, when all DM is in non-condensated form, to ρ_χ^{BEC} when all DM is condensated, when the transition phase ends. So we introduce the volume fraction of matter in the BEC phase [14]

$$f(t) = \frac{\rho_\chi(t) - \rho_\chi^{norm}}{\rho_\chi^{BEC} - \rho_\chi^{norm}} \quad , \tag{25}$$

and we can write

$$\rho_\chi(t) = \rho_\chi^{norm} \left(1 + n_\chi f(t)\right) \quad , \tag{26}$$

$$n_\chi = \frac{\rho_\chi^{BEC} - \rho_\chi^{norm}}{\rho_\chi^{norm}} \quad . \tag{27}$$

When the condensation starts, at $t = t_{crt}$ all the DM is in non-condensated form, i.e., $\rho_\chi(t_{crt}) = \rho_\chi^{norm}$ and $f(t_{crt}) = 0$. At the end of the condensation all the DM is in BEC form, $\rho_\chi(t_{BEC}) = \rho_\chi^{BEC}$, and $f(t_{BEC}) = 1$. With help of the energy conservation equation (7) we find that [14]

$$a(t) = a_{crt} \left(1 + r f(t)\right)^{-1/3} \quad , \tag{28}$$

$$r = \frac{n_\chi}{1 + P_{crt}/\rho_\chi^{norm} c^2} \quad , \tag{29}$$

where $a(t_{crt}) = a_{crt}$ and the phase transition ends at

$$1 + z_{BEC} = (1 + r)^{1/3}(1 + z_{crt}) \quad . \tag{30}$$

Using the Friedmann's equation (5) we find that the evolution of the volume fraction is given by

$$\frac{df}{d\tau} = -3 \left(\frac{1 + rf}{r} \right) \sqrt{ \frac{\Omega_b^{(0)}}{(a_{crt}/a_0)^3}(1 + rf) + \frac{\Omega_r^{(0)}}{(a_{crt}/a_0)^4}(1 + rf)^{4/3} + \Omega_\chi^{(0)}(1 + n_\chi f) + \Omega_\Lambda } \quad , \tag{31}$$

where $\Omega_\chi^{(0)} = \rho_\chi^{norm}/\rho_{crt}^{(0)}$ and $\tau = H_0 t$. Since $\rho_\chi^{BEC} < \rho_\chi^{norm}$ we will have $r \in (-1, 0)$ and $n_\chi < 0$. On the other hand $P_{crt}/\rho_\chi^{norm} c^2 = \sigma^2 << 1$ and from equation (28) we can approximate $r \approx n_\chi$ [14] and if we neglect the radiation contribution to the energy density the equation (31) has the solution

$$f(t) = \frac{\Omega_\Lambda^2}{r\Omega_g}\left[\frac{1 + \Omega_c e^{-3(\Omega_\Lambda/H_0^{-1})(t-t_{crt})}}{1 - \Omega_c e^{-3(\Omega_\Lambda/H_0^{-1})(t-t_{crt})}}\right]^2 - \frac{\Omega_\Lambda + \Omega_g}{r\Omega_g} \quad , \tag{32}$$

where H_0^{-1} is the Hubble time and

$$\Omega_g = \frac{\Omega_b^{(0)}}{(a_{crt}/a_0)^3} + \Omega_\chi^{(0)} \quad , \tag{33}$$

$$\Omega_c = \frac{\sqrt{\Omega_g + \Omega_\Lambda} - \Omega_\Lambda}{\sqrt{\Omega_g + \Omega_\Lambda} + \Omega_\Lambda} \quad . \tag{34}$$

If we neglect both radiation and cosmological constant contribution to the total energy density it is straightforward to find

$$f(t) = \frac{1}{r}\left(1 + \frac{2}{3}H_0\sqrt{\Omega_g}(t - t_{crt})\right)^{-2} - \frac{1}{r} \quad . \tag{35}$$

When the condensation process ends, for $t \geq t_{BEC}$, the evolution of the Universe is given by equation (20). As can be seen from the equations (32) and (35) the scale factor do not depend on r.

With equations (28), (32) and (35) we calculate the Hubble parameter at the critical point for the model with and without cosmological constant, respectively

$$H_{crt} = \frac{4}{9}H_0\sqrt{\Omega_b^{(0)}(1 + z_{crt})^3 + \Omega_\chi^{(0)}} \quad , \tag{36}$$

$$H_{crt} = H_0\sqrt{\Omega_b^{(0)}(1 + z_{crt})^3 + \Omega_\chi^{(0)} + \Omega_\Lambda} \quad , \tag{37}$$

which allow us to constraint the critical redshift z_{crt}, since the Hubble parameter at the critical point must be bigger than the one today. In Figure (1) we see the ratio between the Hubble parameter at the beginning of the condensation process and the Hubble parameter today as a fuction of the critical redshift z_{crt}. The requirement that $H_{crt}/H_0 > 1$ is satisfied for $z_{crt} > 3.5$, $z_{crt} > 3.75$ and $z_{crt} > 0$ respectively.

The effective equation of state parameter ω is

$$\omega = \frac{p}{\rho c^2} = \frac{\sigma^2 \Omega_\chi^{(0)}(1 + z_{crt})^{-3} - \Omega_\Lambda R^3}{\Omega_b^{(0)} + \Omega_\chi^{(0)}(1 + z_{crt})^{-3} + \Omega_\Lambda R^3} \quad , \tag{38}$$

where $R = a/a_0$. This expression and the conservation equation (7) allow us to calculate the speed of sound $c_s^2 = \partial p/\partial \rho$, that is

$$\frac{c_s^2}{c^2} = \frac{\sigma^2 \Omega_\chi^{(0)}(1 + z_{crt})^{-3}}{\Omega_b^{(0)} + \Omega_\chi^{(0)}(1 + z_{crt})^{-3} + \Omega_\Lambda R^3} \quad , \tag{39}$$

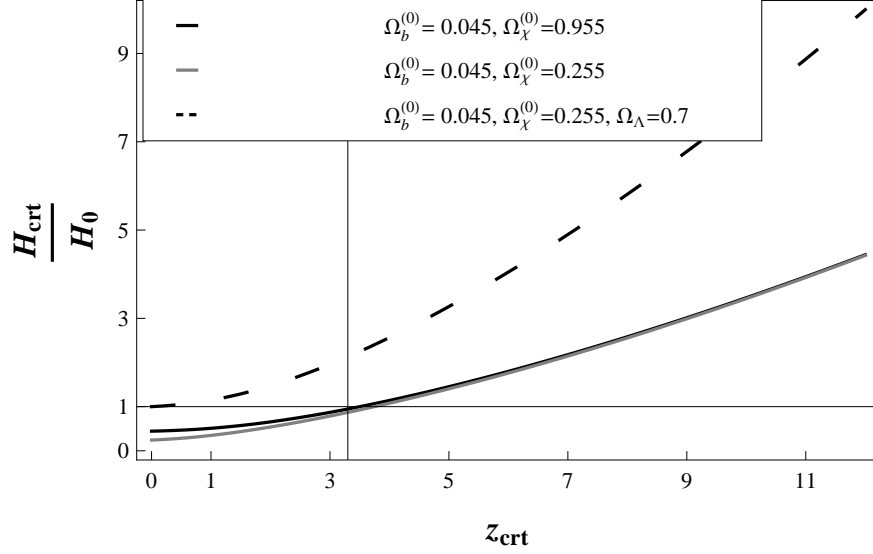which we can approximate to null, since $\sigma^2 << 1$.

Figure 1: The ratio between the Hubble parameter at the beginning of the condensation and the Hubble parameter today for a Universe filled with cosmological constant (dashed line) and without cosmological constant (solid lines).

3   Cosmological density perturbations

There are other studies about this subject: with Newtonian gravity [20], with the new-Newtonian gravity [21, 22], with Post-Newtonian approximation by using the conservation of the general relativistic energy-momentum tensor, and considering the small velocity limit [23] and the relativistic case [24] where the model unifies the dark matter and dark energy.

Here we study the evolution of the structure formation during the BEC phase transition using the gauge-invariant formalism [25, 26] for the classical linear perturbations. Whe choose the longitudinal (or conformal-Newtonian) gauge, in which for a diagonal energy-momentum tensor the perturbed metric is

$$ds^2 = a(\eta)^2 \left[(1 + 2\Phi)d\eta^2 - (1 - 2\Phi)d\vec{x}^2\right] \quad , \tag{40}$$

where Φ is the gauge-invariant scalar perturbation, which can be interpreted as the relativistic generalization of the Newtonian potential and η is the conformal time, related to the cosmic time t by $cdt = ad\eta$. We replace the metric (40) in the Einstein field equations (3) and keeping only the first order terms we get the equations of motion for the scalar potential Φ

$$\nabla^2\Phi - 3\mathcal{H}\left(\Phi' + \mathcal{H}\Phi\right) = 4\pi a^2 G \delta\rho \quad , \tag{41}$$

$$\Phi'' + 3\mathcal{H}\Phi' + \left(2\mathcal{H}' + \mathcal{H}^2\right)\Phi = 4\pi G a^2 \delta p/c^2 \quad , \tag{42}$$

where the apostrophe $'$ means the derivative to the conformal time η, $\mathcal{H} = a'/a$ is the conformal Hubble parameter and $\delta\rho$ and δp are the gauge-invariant perturbations of the total density and the total pressure, respectively.

For the case of adiabatic perturbation [25, 26] we use the relation

$$\delta p = c_s^2 \delta\rho \quad , \tag{43}$$

8

combined with the equations (41), where $c_s^2 = \partial p / \partial \rho$ is the fluid sound speed, to find

$$\Phi'' + 3\mathcal{H}\left(1 + \frac{c_s^2}{c^2}\right)\Phi' + \left[2\mathcal{H}' + \mathcal{H}^2\left(1 + 3\frac{c_s^2}{c^2}\right) + \frac{c_s^2}{c^2}k^2\right]\Phi = 0 \quad , \tag{44}$$

where k^2 is the wave-number and we made $\nabla^2 \Phi = -k^2 \Phi$. With help of a change of variable [26] and the background equations we find

$$a^2 \frac{d^2\Phi}{da^2} + \left[4 + 3\frac{c_s^2}{c^2} - \frac{1}{2}\left(1 + 3\frac{p}{\rho c^2}\right)\right]a\frac{d\Phi}{da} + \left[1 + 3\frac{c_s^2}{c^2}\left(1 + \frac{k^2}{8\pi G\rho a^2}\right) - \frac{3}{2}\left(1 + 3\frac{p}{\rho c^2}\right)\right]\Phi = 0 \quad . \tag{45}$$

To examine the density perturbations $\delta\rho$ we calculate the relations $T^{\mu\nu}_{;\nu} = 0$ up to first order, where $;\nu$ is the covariant derivative, to find [25, 26]

$$\delta\rho' + \mathcal{H}(\delta\rho + \delta p) - 3\Phi'(\rho + p) + a(\rho + p)\delta u^i_{,i} = 0 \quad , \tag{46}$$

$$a^{-4}\left[a^5(\rho + p)\delta u^i_{,i}\right]' + \nabla^2 \delta p + (\rho + p)\nabla^2 \Phi = 0 \quad , \tag{47}$$

where δu^i is the fluid velocity perturbation and $,i$ is the ordinary spatial derivative.

In this analysis the baryons and DM do not interact except through gravity and we neglect the radiation contribution. So each component satisfy the pair of equations (46) and (47) separately. Since $P_{crt}/\rho_\chi c^2 = \sigma^2 << 1$ is true we can make an approximation to find that both baryons and DM satisfy the same differential equation

$$(\delta - 3\Phi)' \quad + \quad a\delta u^i_{,i} = 0 \quad , \tag{48}$$

$$\delta = \frac{\delta\rho_b}{\rho_b} \quad = \quad \frac{\delta\rho_\chi}{\rho_\chi} \quad , \tag{49}$$

where δ is the density contrast.

Combining equations (47) and (48) and making a change of variables for $R = a/a_0$ we find that

$$\frac{d}{dR}\left[R^2\mathcal{H}\frac{d}{dR}(\delta - 3\Phi)\right] - \frac{\nabla^2\Phi}{\mathcal{H}} = 0 \quad . \tag{50}$$

From equation (38) we see that

$$\omega \approx \begin{cases} 0 & , \quad \Omega_\Lambda = 0 \quad , \\ \frac{-\Omega_\Lambda R^3}{\Omega_m + \Omega_\Lambda R^3} & , \quad \Omega_\Lambda \neq 0 \end{cases} \quad , \tag{51}$$

where $\Omega_m = \Omega_b^{(0)} + \Omega_\chi^{(0)}(1 + z_{crt})^{-3}$. From equation (39) for the sound speed we have

$$\frac{c_s^2}{c^2} \approx 0 \quad , \tag{52}$$

since we consider $\sigma^2 << 1$.

3.1 Universe filled with baryons and DM

With equations (74) and (52) the perturbation equation (45) will be

$$R^2\frac{d^2\Phi}{dR^2} + \frac{7}{2}R\frac{d\Phi}{dR} - \frac{1}{2}\Phi = 0 \quad , \tag{53}$$

9

which has a simple power law solution

$$\Phi \;\;=\;\; C_+ R^{n+} + C_- R^{n-} \quad, \tag{54}$$

$$n_\pm \;\;=\;\; \frac{-5 \pm \sqrt{33}}{4} \quad, \tag{55}$$

and the growing and decaying modes of the potential Φ as a function of time are shown in Figure (2).

Applying the solution (54) to the equation (50) it is straightforward to find

$$\delta = 3\Phi(R) + \frac{k^2}{a_0^2 H_0^2 \Omega_m}\left[C_+ \frac{R^{(n_+ - 2)}}{n_+(n_+ - 2)} + C_- \frac{R^{(n_- - 2)}}{n_-(n_- - 2)} \right] \quad. \tag{56}$$

The growing and decaying modes of the matter density contrast as a function of time are shown in Figures (3) and (4).
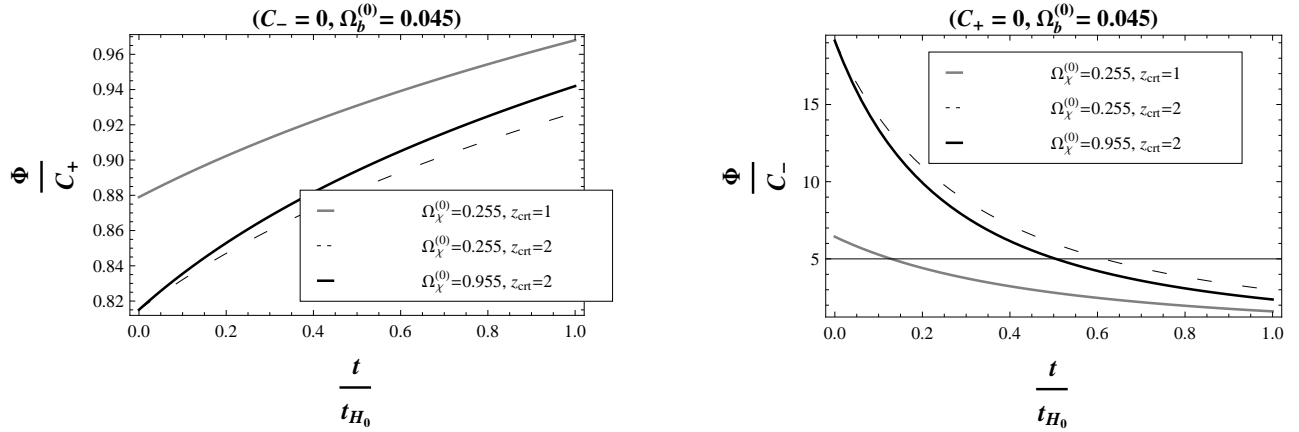
Figure 2: Growing and decaying modes of the potential Φ as a function of time in a Universe filled with baryons and DM.

3.2 Universe filled with baryons, DM and cosmological constant

In the case of Universe dominated by a cosmological constant with density parameter Ω_Λ we find, from equations (45), (51) and (52), that

$$R^2 \frac{d^2\Phi}{dR^2} + \left[4 - \frac{1}{2}\left(1 - 3\frac{\Omega_\Lambda R^3}{\Omega_m + \Omega_\Lambda R^3}\right)\right] R\frac{d\Phi}{dR} + \left[1 - \frac{3}{2}\left(1 - 3\frac{\Omega_\Lambda R^3}{\Omega_m + \Omega_\Lambda R^3}\right)\right] R = 0 \quad, \tag{57}$$

and introducing the new variables

$$x \;\;=\;\; -\frac{\Omega_\Lambda R^3}{\Omega_m + \Omega_\Lambda R^3} \quad, \tag{58}$$

$$\Phi \;\;=\;\; x^{\nu_\pm} y(x) \quad, \tag{59}$$

$$\tag{60}$$

we will have

$$x(1-x)\frac{d^2y}{dx^2} + \left[c_\pm - (a_\pm + b_\pm + 1)x\right]\frac{dy}{dx} - (a_\pm b_\pm)\,y = 0 \quad, \tag{61}$$
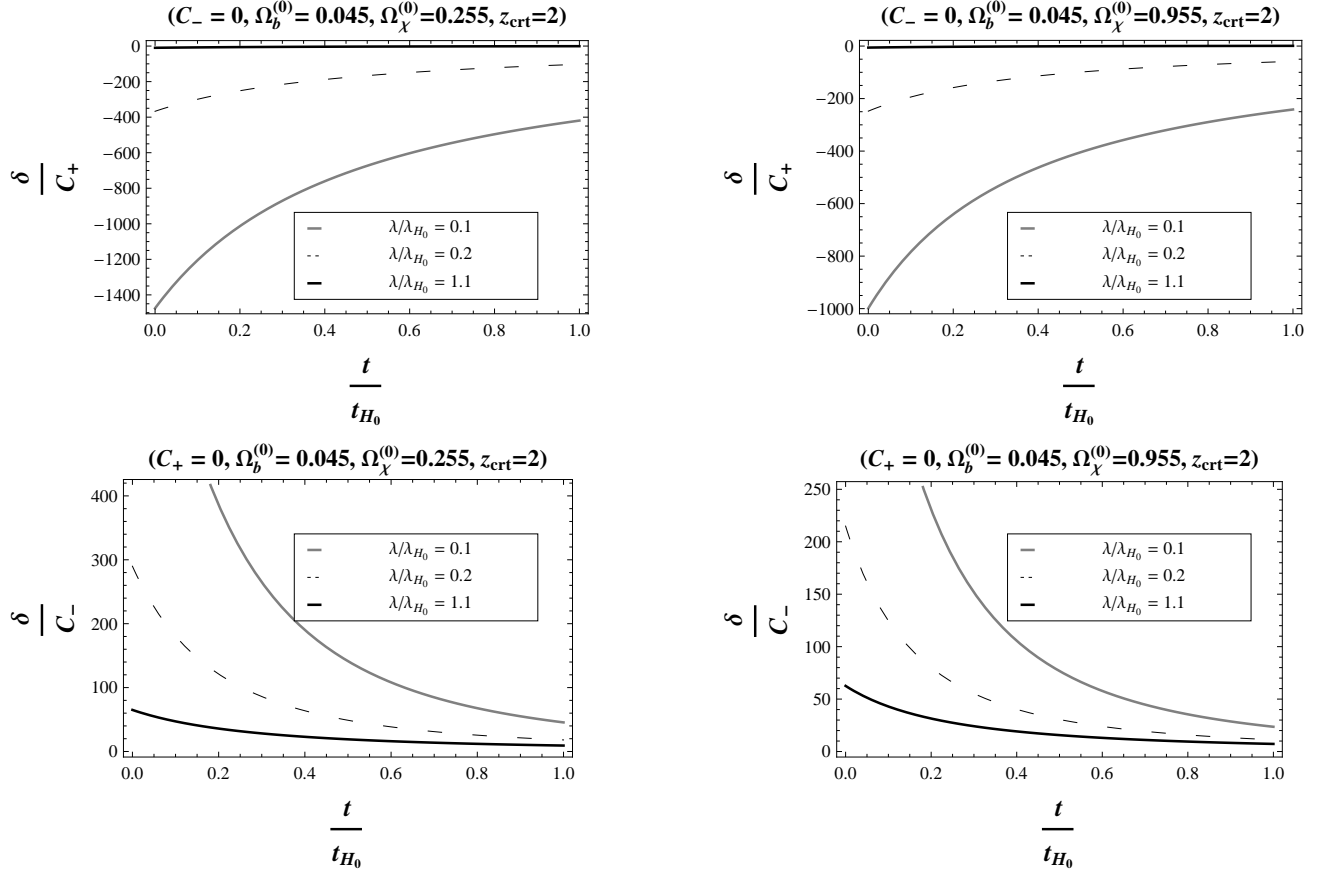
Figure 3: Growing and decaying modes of the matter density contrast as a function of time for various wavelengths in a Universe filled with baryons and DM, where $\lambda_{H_0} = cH_0^{-1}$ is the Hubble wavelength.

with

$$\nu_\pm = \frac{-5 \pm \sqrt{33}}{12} \quad , \tag{62}$$

$$a_\pm = \frac{1}{4} \pm \frac{\sqrt{33}}{12} \quad , \tag{63}$$

$$b_\pm = a_\pm \quad , \tag{64}$$

$$c_\pm = 1 \pm \frac{\sqrt{33}}{6} \quad . \tag{65}$$

The solution to the equation (57) is

$$\Phi(R) = C_+ x^{\nu_+} \, _2F_1(a_+, b_+; c_+; x) + C_- x^{\nu_-} \, _2F_1(a_-, b_-; c_-; x) \quad , \tag{66}$$

where $_2F_1(a, b; c; x)$ is the hypergeometric function. The growing and decaying modes of the potential Φ as a function of time are shown in Figure (5).

In this case we can solve equation (50) numericaly with help of expression (66) to find the density contrast. The growing and decaying modes of the matter density contrast as a function of time are shown in Figure (6).
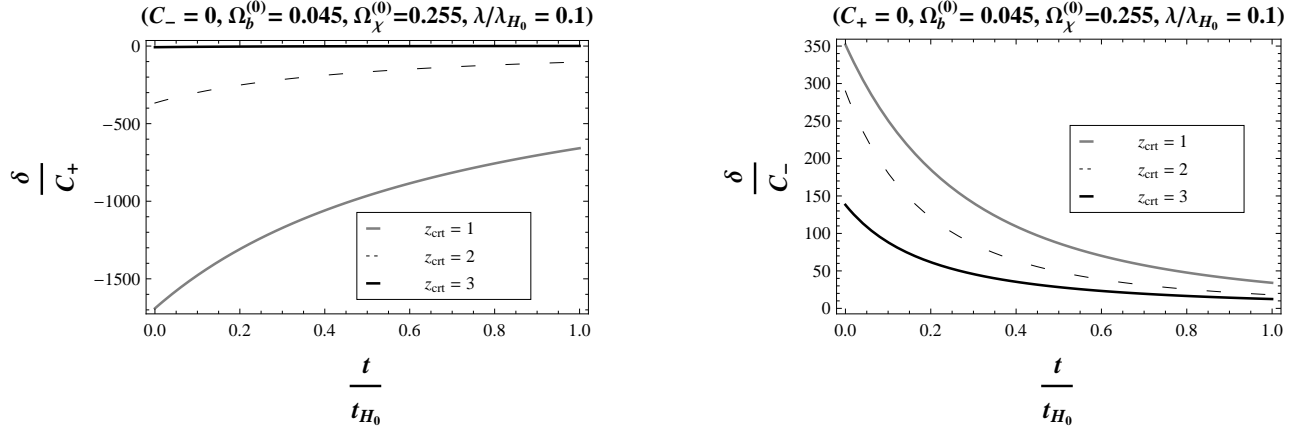
11

Figure 4: Growing and decaying modes of the matter density contrast as a function of time for various critical redshifts in a Universe filled with baryons and DM.
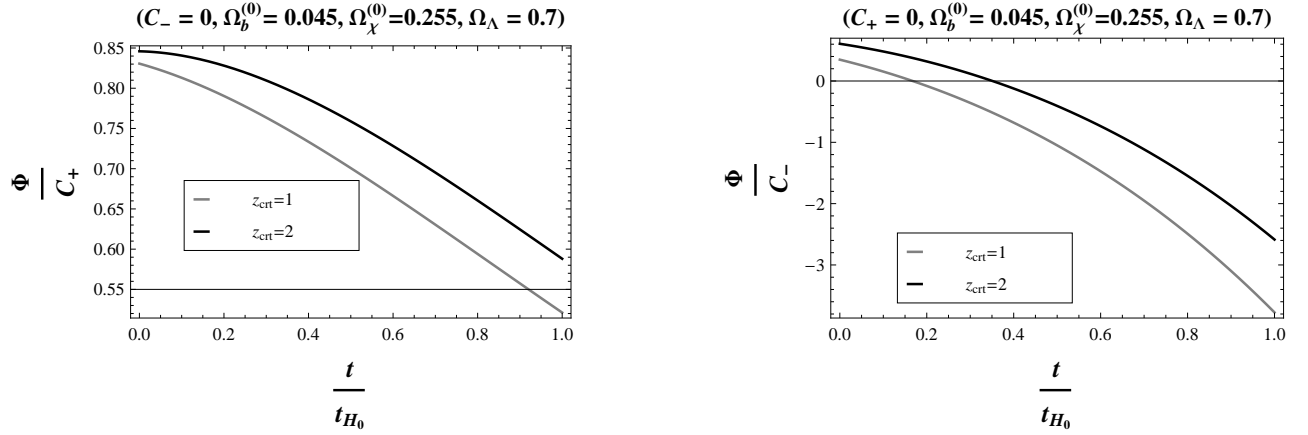

Figure 5: Growing and decaying modes of the potential Φ as a function of time in a Universe filled with baryons, DM and cosmological constant.

4 Cosmological gravitational waves

In this section we study the gravitational waves generated by the expansion of the Universe during the phase transition of normal DM into BEC DM. To do so it is necessary to perturb to first order the metric tensor and we write

$$ds^2 = c^2 dt^2 - \left[a(t)\delta_{ij} + h_{ij}(t,\vec{x})\right] dx^i dx^j \quad , \tag{67}$$

where $h_{ij}(t,\vec{x})$ are the metric perturbations and we adopt the transverse and traceless (TT) gauge [25, 27, 28], i.e., $h_i{}^i = 0$ and $h_i{}^j{}_{,j} = 0$. Applying the metric (67) to the Einstein field equations (3) keeping only the first order terms [25] and with help of the equations of motion (5) we find the dynamics equations of the cosmological gravitational waves

$$\ddot{h}_{ij} - \frac{\dot{a}}{a}h_{ij} + \left(-\frac{\nabla^2}{a^2} - \frac{\ddot{a}}{a}\right)h_{ij} = 0 \quad , \tag{68}$$
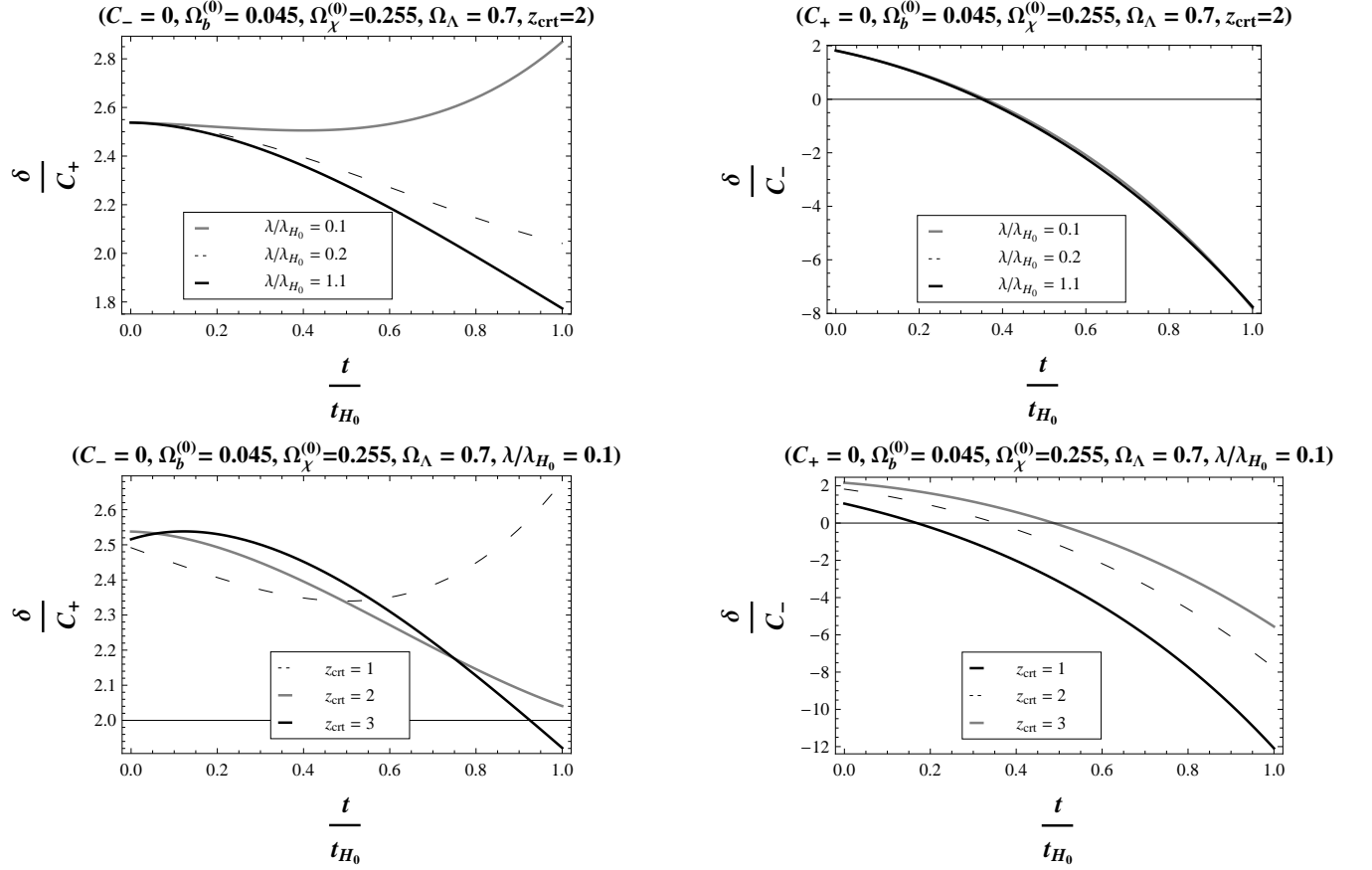
12

Figure 6: Growing and decaying modes of the matter density contrast as a function of time for various parameters in a Universe filled with baryons, DM and cosmological constant, where $\lambda_{H_0} = cH_0^{-1}$ is the Hubble wavelength.

where ∇^2 is the Laplace operator. We introduce the expansion

$$h_{ij}(t, \vec{x}) = \sum_{s=\otimes,\oplus} \int \frac{d^3k}{(2\pi)^{3/2}} e^{-i\vec{k}\cdot\vec{x}} h^{(s)}(t, \vec{k}) \epsilon_{ij}^{(s)}(\vec{k}) \quad , \tag{69}$$

where $s = \otimes, \oplus$ are the two polarization states of the gravitational waves, $\vec{k}$ is the wave-number and $\epsilon_{ij}^{(s)}(\vec{k})$ is the polarization tensor, where $\epsilon_{ij}^{(s)}(\vec{k})\epsilon^{(s')ij}(\vec{k'}) = 2\delta_{ss'}\delta^3(\vec{k} - \vec{k'})$ and $h^{(\otimes)} = h^{(\oplus)} = h(t, \vec{k})$. Writing the wave equation (68) as a function of the scale factor a, using the (69) and the background equations (5) we find

$$\frac{d^2h}{da^2} - \frac{3}{2a}(1+\omega)\frac{dh}{da} + \left[\frac{3k^2}{8\pi G\rho a^4} + \frac{1}{2a^2}(1+3\omega)\right]h = 0 \quad , \tag{70}$$

with p and ρ being the total pressure and density respectively.

With the solution of the wave equation (70) we can calculate de gravitational waves energy density, that is related to the perturbation $h_{ij}(t, \vec{x})$ by [27]

$$\rho_{gw} = \frac{1}{32\pi G}\left\langle \dot{h}_{ij}\dot{h}^{ij}\right\rangle \quad , \tag{71}$$

13

where $\langle \cdots \rangle$ means spatial average over many reduced wavelengths. The logarithmic energy density is [27]

$$\Omega_{gw} = \frac{8\pi G}{3H_0^2} \frac{d\rho_{gw}}{d\ln k} \quad , \tag{72}$$

and the spectral density $S_h(k)$ [27] is related to the logarithmic energy density by

$$\Omega_{gw} = \frac{4\pi^2}{3H_0^2} k^3 S_h(k) \quad . \tag{73}$$

4.1 Universe filled with baryons and DM

In order to find a solution to the wave equation (70) we first consider a Universe filled with baryons, which are pressurless, and DM in phase transition. Since $\sigma^2 << 1$ and, in this case, $\Omega_\Lambda = 0$, we can make the following approximations

$$\omega \approx 0 \quad , \tag{74}$$

$$r \approx n_\chi \quad . \tag{75}$$

And the wave equation (70) will be written as

$$R^2 \frac{d^2 h}{dR^2} - \frac{3}{2} R \frac{dh}{dR} + \left(\frac{k^2}{a_0^2 H_0^2 \Omega_m} R + \frac{1}{2} \right) h = 0 \quad , \tag{76}$$

where $R = a/a_0$ and

$$\Omega_m = \Omega_b^{(0)} + \Omega_\chi^{(0)} (1 + z_{crt})^{-3} \quad . \tag{77}$$

To solve the above differential equation for the gravitational waves it is necessary to introduce the variables

$$q^2 \equiv \frac{k^2}{a_0^2 H_0^2 \Omega_m} \quad , \tag{78}$$

$$x^2 \equiv 4Rq^2 \quad , \tag{79}$$

$$h \equiv x^{5/2} y(x) \quad , \tag{80}$$

and we obtain the Bessel differential equation

$$x^2 y''(x) + xy'(x) + (x^2 - 17/4) y(x) = 0 \quad , \tag{81}$$

which gives the solution

$$h(2q\sqrt{R}) = \left(2q\sqrt{R} \right)^{5/2} \left(C_1 J_\nu(2q\sqrt{R}) + C_2 J_{-\nu}(2q\sqrt{R}) \right) \quad , \tag{82}$$

where $J_\nu(x)$ is the Bessel function of first kind with order $\nu = \sqrt{17}/2$ and C_1 and C_2 are integration constants. In Figure (7) we show the logarithmic energy density for several values of the parameters for a Universe filled with baryons and CDM during the transition phase as a fuction of the ratio between the gravitational waves wavelength and the Hubble wavelength, defined as $\lambda_{H_0} = cH_0^{-1}$.
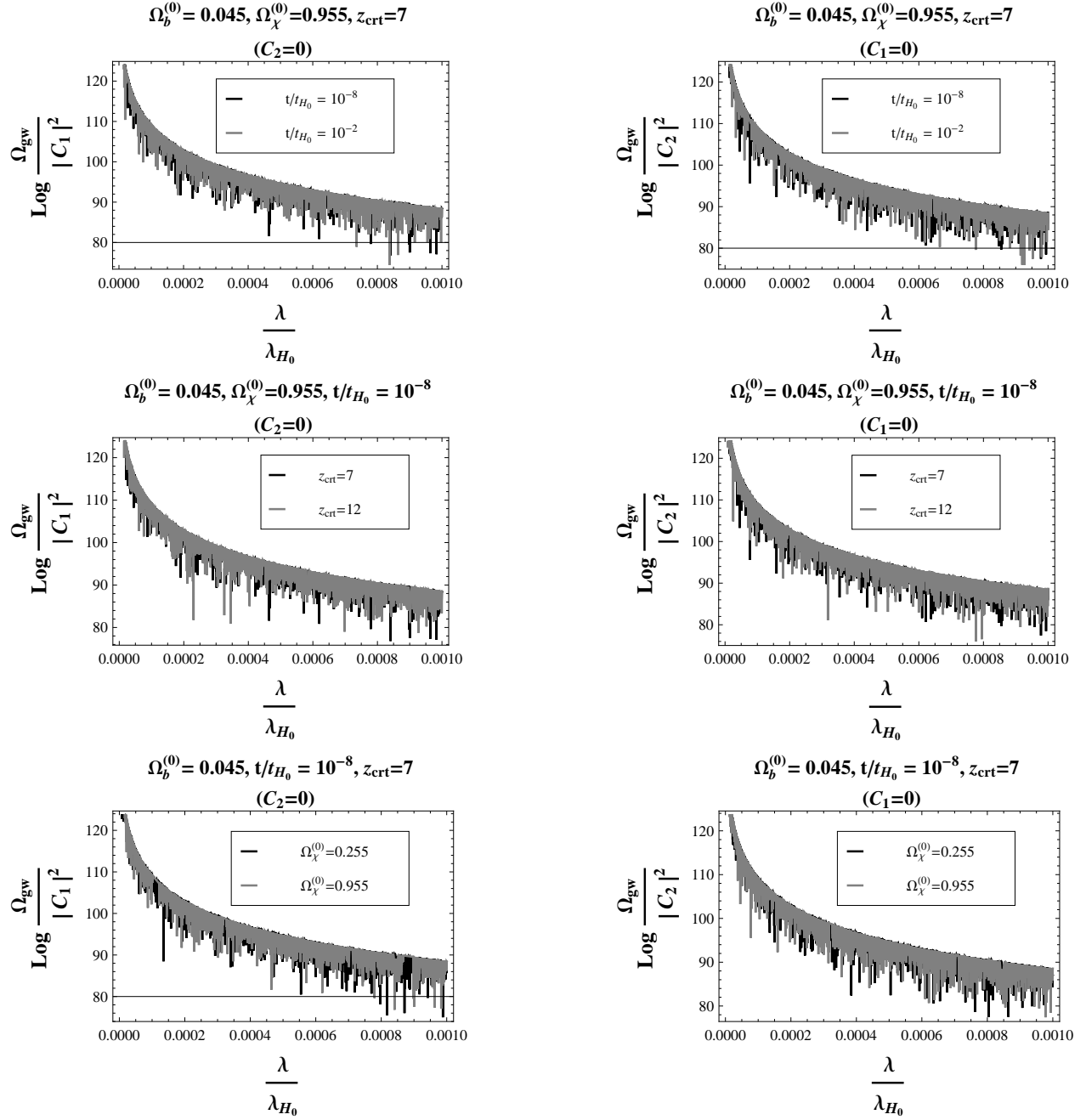
Figure 7: Gravitational waves energy density as a fuction of the wavelength for fixed time in the case of a Universe filled with baryons and DM during the phase transition, where $\lambda_{H_0} = cH_0^{-1}$ is the Hubble wavelength.

4.2 Universe filled with baryons, DM and cosmological constant

In this section we will consider a Universe filled with baryons, DM in phase transition and a cosmological constant. In the equation of state parameter (38) above the first term can be discharged, since

σ^2 is very small and we can make the approximation

$$\omega \approx -\frac{\Omega_\Lambda R^3}{\Omega_m + \Omega_\Lambda R^3} \quad , \tag{83}$$

where

$$\Omega_m = \Omega_b^0 + \Omega_\chi^{(0)}(1 + z_{crt})^{-3} \quad , \tag{84}$$

and the wave equation (70) is written as

$$R^2\frac{d^2h}{dR^2} - \frac{3}{2}R\left(\frac{\Omega_m}{\Omega_m + \Omega_\Lambda R^3}\right)\frac{dh}{dR} + \left(\frac{q^2 R}{\Omega_m + \Omega_\Lambda R^3} + \frac{1}{2}\left(1 - 3\frac{\Omega_\Lambda R^3}{\Omega_m + \Omega_\Lambda R^3}\right)\right)h = 0 \quad , \tag{85}$$

with

$$q^2 = \frac{k^2}{H_0^2 a_0^2} \quad . \tag{86}$$

For the case of perturbations with wavelength bigger than the Hubble length we approximate $q \approx 0$ to find

$$R^2\frac{d^2h}{dR^2} - \frac{3}{2}R\left(\frac{\Omega_m}{\Omega_m + \Omega_\Lambda R^3}\right)\frac{dh}{dR} + \frac{1}{2}\left(1 - 3\frac{\Omega_\Lambda R^3}{\Omega_m + \Omega_\Lambda R^3}\right)h = 0 \quad , \tag{87}$$

and we introduce the variables

$$x = -\frac{\Omega_\Lambda}{\Omega_m}R^3 \quad , \tag{88}$$

$$h(x) = x^{\nu_\pm}y(x) \quad , \tag{89}$$

where $\nu_\pm = 5/12 \pm \sqrt{17}/12$ and we find

$$x(1 - x)\frac{d^2y}{dx^2} + [c_\pm - (a_\pm + b_\pm + 1)x]\frac{dy}{dx} - a_\pm b_\pm y = 0 \quad , \tag{90}$$

with

$$a_\pm = \frac{(3 - 2\sqrt{5} \pm \sqrt{17})}{12} \quad , \tag{91}$$

$$b_\pm = \frac{(3 + 2\sqrt{5} \pm \sqrt{17})}{12} \quad , \tag{92}$$

$$c_\pm = 1 \pm \frac{\sqrt{17}}{6} \quad . \tag{93}$$

The solution is found to be

$$h(R) = C_+ x^{\nu_+}{}_2F_1(a_+, b_+; c_+; x) + C_- x^{\nu_-}{}_2F_1(a_-, b_-; c_-; x) \quad , \tag{94}$$

where $_2F_1(a, b; c; x)$ is the hypergeometric function.

For small wavelenghts, the equation (85) will be

$$R^2\frac{d^2h}{dR^2} - \frac{3}{2}R\left(\frac{\Omega_m}{\Omega_m + \Omega_\Lambda R^3}\right)\frac{dh}{dR} + \frac{q^2 R}{\Omega_m + \Omega_\Lambda R^3}h = 0 \quad , \tag{95}$$

which can only be solved numericaly. In Figure (8) we show the spectral density, that in this case does not depend on the wavelength, as a function of time in a Universe filled with baryons CDM and cosmological constant during the transition.
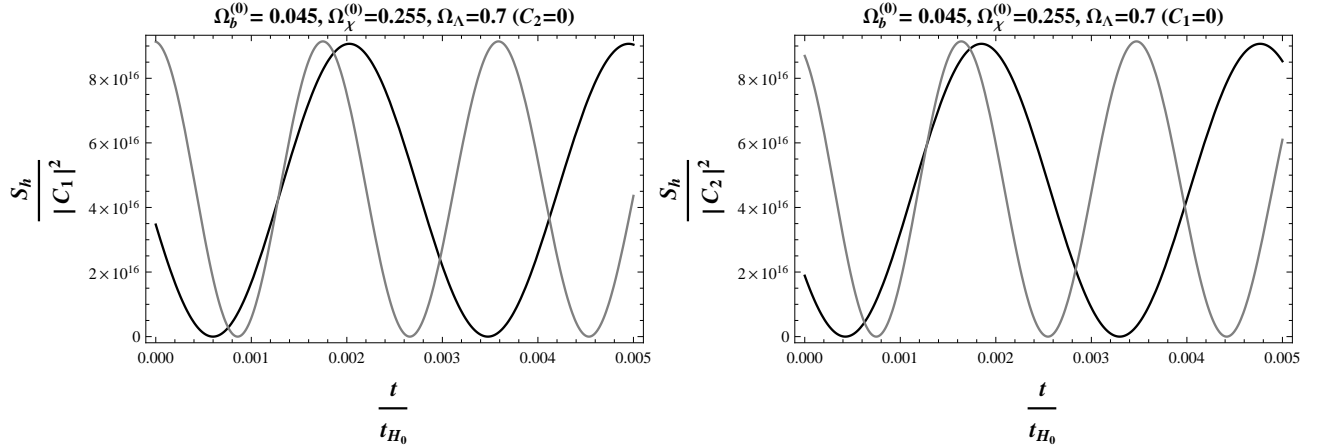
16

Figure 8: Gravitational waves spectral density as a fuction of time in the case of a Universe filled with baryons and DM during the phase transition. The black curve shows the case where $z_{crt} = 7$ and in the gray one $z_{crt} = 12$.

5 Conclusions

In this letter we considered a Universe where initially the DM is nearly pressureless and after a phase transition the DM is all in form of BEC DM, which has a polytropic equation of state. During the transition phase both non-condensated and condensated DM coexist and the time evolution of the Universe expansion is changed. We studied the scalar and tensorial perturbations generated by the background expansion during the phase transition in two cases: with and without cosmological constant.

Firstly, we verify from Figure 1 and from equations (36, 37) that the ratio between the Hubble parameter at the beginning of the condensation process and the Hubble parameter today, with the condition $H_{crt}/H_0 > 1$, is satisfied for $z_{crt} > 3.5$, $z_{crt} > 3.75$ and $z_{crt} > 0$, what was expected. Two situation are analysed: for a Universe filled with cosmological constant and without cosmological constant. We can see that the case where Λ is present is favored in the sense that the ratio of the Hubble parameters is bigger than the case where the cosmologcial constant is absent. This indicates that in a Universe whose material content is baryonic matter, BEC DM and cosmological constant the condensation can begin later when compared with the model without Λ.

When we analyze the scalar potential Φ we see that in the case without cosmological constant, Figure 2, there is a growing and a decaying mode. Both modes are more sensitivy to changes in the amount of DM than to changes in the critical redshift z_{crt}. In the Universe with cosmological constant, Figure 5, we see two decaying modes and smaller the critical redshfit z_{crt} smaller the potential Φ.

In Figures 3 and Figure 4 we see that the density contrast amplitude, in a Universe without cosmological constant, is sensitivy to the amount of DM and bigger the perturbation wavelength bigger the perturbation in the case of the growing modes and smaller in the case of the decaying modes. The same behavior can be seen when the critical redshfit is varied.

For a Universe with cosmological constant the contrast amplitude in Figure 6 can not be distinguished when we vary the perturbations wavelength in the case of the decaying modes. For the growing modes the amplitude initially decays but after some time interval it grows again. The decaying period of the growing modes depends on the wavelength. The bigger the wavelength bigger is the decaying time interval. The same behavior is reproduced when we vary the critical redshift z_{crt}.

17

In the case of tensorial perturbations in a Universe without cosmological constant we see in Figure 7 that the gravitational waves energy density changes little when we vary the time of beginning of the transition (z_{crt}) and even the amount of DM in the Universe. Figure 8 shows little changes in the spectral density in a Universe with cosmological constant when we change the beginning of the transition.

The Bose-Einstein condensated dark matter model can be an interesting theory to explain the nature of dark matter in the Universe. A more detailed investigation of the model discussed herein, including the generalization of the equation of state of BEC dark mater model, is being made as well as the bayesian analysis to determine the most probable values of the various parameters of the theory. The BEC DM model still has to pass by several observational tests to prove as a viable cosmological model. Its importance grows as weakly interacting massive particles, or WIMPs, are searched in the Large Hadron Collider or other ground-based experiments.

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